

AD-A071 377

OKLAHOMA UNIV NORMAN SCHOOL OF AEROSPACE MECHANICAL --ETC F/G 11/4  
ANALYSES OF PLATES CONSTRUCTED OF FIBER-REINFORCED BIMODULUS CO--ETC(U)  
JUN 79 J N REDDY, C W BERT

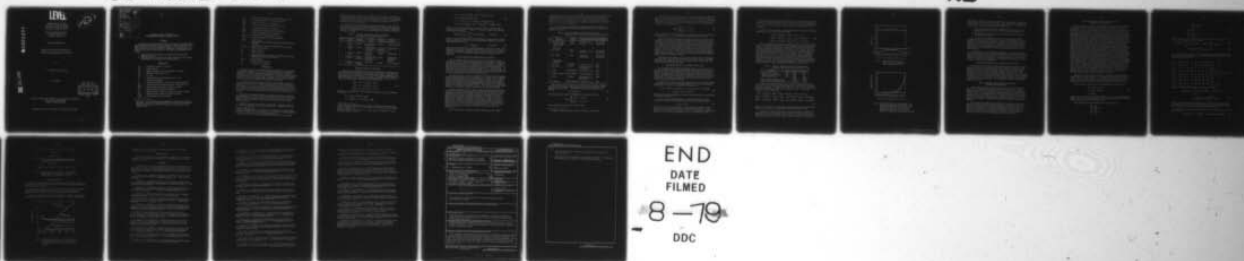
N00014-78-C-0647

UNCLASSIFIED

OU-AMNE-79-8

NL

1 OF 1  
AD-A071377



END

DATE  
FILMED

8-79

DDC

# LEVEL II

12

Department of the Navy  
OFFICE OF NAVAL RESEARCH  
Structural Mechanics Program  
Arlington, Virginia 22217

Contract N00014-78-C-0647  
Project NR 064-609  
Technical Report No. 2

Report OU-AMNE-79-8

ANALYSES OF PLATES CONSTRUCTED OF  
FIBER-REINFORCED BIMODULUS MATERIALS

*See 1473 in Reels*

by

J. N. Reddy and C. W. Bert

June 1979

DDC  
RECEIVED  
JUL 19 1979  
D

School of Aerospace, Mechanical and Nuclear Engineering  
University of Oklahoma  
Norman, Oklahoma 73019

Approved for public release; distribution unlimited

79 07 17 024

ADA071377

DDC FILE COPY

Accession For	
NTIS GRA&I	<input checked="" type="checkbox"/>
DDC TAB	<input type="checkbox"/>
Unannounced	<input type="checkbox"/>
Justification	
By	
Distribution/	
Availability Codes	
Dist	Availand/or special
A	

-2-

# ANALYSES OF PLATES CONSTRUCTED OF FIBER-REINFORCED BIMODULUS COMPOSITE MATERIAL \*

## ABSTRACT

To implement the structural application of the recently introduced fiber-governed symmetric compliance model for bimodulus composite materials, both classical closed-form and finite-element solutions are being developed. This paper summarizes the results obtained for deflection of single-layer orthotropic and two-layer, cross-ply plates of the following configurations and loadings:

1. Thin elliptic plates clamped on the boundary and subjected to uniform pressure.
2. Moderately thick rectangular plate freely supported on the boundary and subjected to sinusoidally distributed pressure.

## NOMENCLATURE

$A_{ij}$	= stretching stiffness ( $i,j=1,2,6$ )
$a,b$	= plate semi-axes
$B_{ij}$	= bending-stretching coupling stiffness ( $i,j=1,2,6$ )
$D_{ij}$	= bending stiffness ( $i,j=1,2,6$ )
$d_x$	= $\alpha(\ )/\alpha x$
$F_i$	= finite-element force components ( $i=1,2,\dots,5$ )
$h$	= total plate thickness
$K_{ij}$	= stiffness coefficients in the finite-element formulation
$L_{\alpha\beta}$	= linear differential operators defined in equations (2)
$M_i, N_i$	= stress couples and stress resultants ( $i=1,2,6$ )
$N_i$	= finite-element interpolation functions
$n, \bar{n}$	= number of nodes per element, number of layers in laminate
$p, p_0$	= normal pressure, intensity of normal pressure
$Q_x, Q_y$	= thickness shear stress resultants

\* This paper is to be presented at the Symposium on Mechanics of Bimodulus Materials, sponsored by the Applied Mechanics Division, at the ASME Winter Annual Meeting, New York, NY, Dec. 1979 and will be published in the symposium volume.

$Q_{ij}$	= plane-stress reduced elastic stiffness ( $i, j=1, 2, 6$ )
$Q_{ijk}$	= $Q_{ij}$ for tension ( $k=1$ ) or compression ( $k=2$ )
$Q_{ijk\ell}$	= $Q_{ijk}$ for layers 1 or 2 ( $\ell=1$ or 2)
$S_{ij}$	= elastic compliance coefficient ( $i, j=1, 2, 6$ )
$S_{ij}^{en}$	= finite-element matrix coefficients
$u, v, w$	= midplane displacements in $x, y, z$ directions
$\bar{u}, \bar{v}$	= total displacements in $x, y$ directions
$u_0, v_0, w_0$	= coefficients in expressions for $u, v, w$
$x, y$	= in-plane rectangular position coordinates
$z_x, z_y$	= $z_{nx}/h, z_{ny}/h$
$z$	= thickness-direction position coordinate, measured from midplane
$z_{nx}, z_{ny}$	= $z$ coordinates of neutral surfaces based on $\epsilon_x$ and $\epsilon_y$
$\epsilon_f$	= fiber-direction strain
$\epsilon_j$	= strain ( $j=1, 2, 6$ )
$\sigma_i$	= stress ( $i=1, 2, 6$ )
$\theta$	= angle between fiber direction of an individual layer and reference direction for the laminate
$\psi_x, \psi_y$	= slope functions
$\Omega$	= middle plane of the plate
$\Omega_e$	= a typical finite element

## INTRODUCTION

An elastic plate is an important structural component in a wide variety of engineering systems. Thus, it is not surprising that the first modern development of the basic constitutive equations of bimodulus<sup>1</sup> materials by Ambartsumyan (1)<sup>2</sup> was followed only one year later by Shapiro's analysis (2) of static deflection of a circular plate constructed of such a material and subjected to a pure radial bending moment. However, in his analysis, he used Love's stress-function formulation rather than plate theory.

Unfortunately, there have been relatively few more recent analyses of bending of bimodulus plates. Notable exceptions are the series of papers by Kamiya (3-5). In (3), he treated large deflections (geometric nonlinearity) of circular plates, using an iterative finite-difference technique, while in (4), he applied the energy method to large deflections of a rectangular plate. The effect of thickness shear deformation on the linear problem was treated in (5).

Ponomarev (6) considered bending of a square plate made of a nonlinear elastic material of a slightly more general nature than bilinear, namely one with a third-degree polynomial stress-strain relation such that the ratio of tension stress to compression stress at the same absolute value of strain remains constant.

All of the analyses mentioned above are limited to isotropic materials with different properties in tension and compression. Apparently, the first

<sup>1</sup> Here a bimodulus material is understood to be one having different elastic properties in tension (T) and compression (C). Materials with same properties in T and C are referred to herein as ordinary materials.

<sup>2</sup> Underlined numbers in parentheses designate References at end of paper.



analysis considering bimodulus materials other than isotropic is due to Jones and Morgan (7), who presented a closed-form solution for cylindrical bending of a thin, cross-ply laminate of finite width but infinite length. They presented numerical results for examples of both antisymmetric and more generally unsymmetric cross-ply constructions.

Apparently, the first analyses of anisotropic bimodulus plates finite in both directions are those recently carried out by the present authors and their colleagues and summarized here. The planform geometries, boundary conditions, loadings, lamination schemes, degree of nonlinearity, and methods of solution are listed in Table 1.

Table 1. Specific Plate Bending Problems Considered For Anisotropic Bimodulus Materials<sup>†</sup>

Problem No.	Planform Geometry	Boundary Conditions	Lamination Scheme	Degree of Nonlinearity	Method of Solution
1	Elliptic thin	Clamped	Unidirectional orthotropic	Linear	Closed form (8)
2	Elliptic thin	Clamped	Unidirectional orthotropic & cross-ply	von Karman geometric nonlinearity	Perturbation (9)
3	Elliptic thin	Clamped	Arbitrary cross-ply	Linear	Closed form
4	Rectangular thick	Freely supported	Unidirectional orthotropic & cross-ply	Linear	Closed form and finite element
5	Rectangular thick	Freely <sup>*</sup> supported	Cross-ply <sup>*</sup>	Linear	Finite element

#### GOVERNING DIFFERENTIAL EQUATIONS

Classical small-deflection theory of thin, unsymmetrically laminated, anisotropic plates was originated by Reissner and Stavsky (10). Apparently, the first display of such a theory for the completely arbitrary anisotropic laminated plate was due to Whitney and Leissa (11). These same equations govern the small deflections of thin, laminated, bimodulus anisotropic plates and thus they are presented here in concise form for completeness:

$$\begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{12} & L_{22} & L_{23} \\ L_{13} & L_{23} & L_{33} \end{bmatrix} \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ p \end{Bmatrix} \quad (1)$$

where the  $L_{\alpha\beta}$  are symmetric linear differential operators ( $L_{\beta\alpha} = L_{\alpha\beta}$ ) defined as follows:

$$\begin{aligned} L_{11} &\equiv A_{11}d_x^2 + 2A_{16}d_xd_y + A_{66}d_y^2 \\ L_{12} &\equiv A_{16}d_x^2 + (A_{12} + A_{66})d_xd_y + A_{26}d_y^2 \end{aligned}$$

\* The program can handle arbitrary lamination arrangement and boundary conditions. However, the results for only cross-ply laminates and freely supported edges are included here.

† Uniform loading is considered in all cases, except in Problem 4, in which sinusoidal loading is used.

$$\begin{aligned}
 L_{13} &\equiv -B_{11}d_x^3 - 3B_{16}d_x^2d_y - (B_{12} + 2B_{66})d_xd_y^2 - B_{26}d_y^3 \\
 L_{22} &\equiv A_{66}d_x^2 + 2A_{26}d_xd_y + A_{22}d_y^2 \\
 L_{23} &\equiv -B_{16}d_x^3 - (B_{12} + 2B_{66})d_x^2d_y - 3B_{26}d_xd_y^2 - B_{22}d_y^3 \\
 L_{33} &\equiv D_{11}d_x^4 + 4D_{16}d_x^3d_y + 2(D_{12} + 2D_{66})d_x^2d_y^2 + 4D_{26}d_xd_y^3 + D_{22}d_y^4
 \end{aligned} \tag{2}$$

Here  $u, v, w$  are the midplane displacements in the  $x, y$  (in-plane) and  $z$  (normal) directions,  $d_x \equiv d(\ )/dx$ ,  $p$  is the normal pressure, and the plate stiffnesses are defined as

$$(A_{ij}, B_{ij}, D_{ij}) = \int_{-h/2}^{h/2} (1, z, z^2) Q_{ij}(z) dz \quad (i, j=1, 2, 6) \tag{3}$$

Here  $h$  is the total thickness of the laminate and  $z$  is measured from the midplane by convention. The  $Q_{ij}(z)$  are the plane-stress reduced elastic stiffnesses defined as follows:

$$\{\sigma_i\} = [Q_{ij}]\{\epsilon_j\} \quad (i, j=1, 2, 6) \tag{4}$$

Here the contracted notation of composite-material theory (12) is used. Thus, subscripts 1 and 2 refer to normal stress (or strain) action in the  $x$  and  $y$  directions and 6 refers to shearing stress (or strain) action with respect to the  $x, y$  axes. Due to thermodynamic considerations, the  $[Q_{ij}]$  matrix is symmetric.

#### PLATE STIFFNESSES FOR BIMODULUS-MATERIAL LAMINATES

Even in laminates made of ordinary materials (those having the same elastic properties in tension and compression), the stiffnesses  $Q_{ij}$  are piecewise-constant functions of the thickness-direction coordinate  $z$ . The individual layers generally consist of unidirectional fiber-reinforced composite material which is orthotropic with respect to its material-symmetry axes: the fiber direction and the two directions orthogonal to it. Thus, when the individual layers are oriented parallel to either the  $x$  or  $y$  axes of the plate, all of the  $A_{ij}$ ,  $B_{ij}$ , and  $D_{ij}$  stiffnesses involving shear-normal coupling (all of those with subscripts 16 and 26) identically vanish.

Furthermore, when a laminate is composed of individual layers of ordinary material arranged symmetrically with respect to its midplane, all of the so-called bending-stretching coupling stiffnesses ( $B_{ij}$ ) vanish. Thus, a single-layer ordinary composite has no  $B_{ij}$  terms present. However, as discussed in (7) and (8), even a single-layer bimodulus composite, as well as a so-called symmetrically laminated multi-layer laminate, has certain  $B_{ij}$  terms present by virtue of the nature of the bimodulus action. Thus, the terms symmetric, antisymmetric, and unsymmetric have no significance for bimodulus laminates.

A popular lamination scheme is the so-called balanced angle-ply laminate. This is one consisting of an even number of layers of identical material and thickness and having an equal number of layers oriented at an angle  $+\theta$  and  $-\theta$  with respect to an arbitrary reference direction. When such a laminate is made of ordinary material, the shear-normal coupling effects ( $Q_{16}$  and  $Q_{26}$  terms) for  $+\theta$  are exactly balanced by the ones at  $-\theta$ . Hence,  $A_{16} = A_{26} = 0$  and the term "balanced". In contrast, if this same laminate is made of bimodulus material and undergoes sufficient bending action that the fiber-direction strains at the top and bottom surfaces of the laminate are opposite in sign, then the terms at  $+\theta$  are not balanced by those at  $-\theta$ . Thus, for this bimodulus laminate,  $A_{16}$  and  $A_{26}$  do not vanish and the term balanced is inappropriate.

Even in the case of a so-called symmetric balanced angle-ply (SBAP)

laminate, such as one having a lamination scheme 0/-0/-0/0, of ordinary composite material, even though  $A_{16}$  and  $A_{26}$  vanish,  $D_{16}$  and  $D_{26}$  do not. A detailed derivation of general expressions for  $D_{16}$  and  $D_{26}$  for such a laminate with an arbitrary even number of layers is given in (13). In view of these considerations, it is clear that for an angle-ply laminate of bimodulus material, there are no vanishing plate stiffnesses.

A listing of all of the plate stiffnesses that vanish for a variety of lamination arrangements of both ordinary and bimodulus composite materials is presented in Table 2.

Table 2. Plate Stiffnesses for Some Popular Lamination Arrangements of Ordinary and Bimodulus Composite Materials

Case	Name of Arrangement	Example Laminate	List of Vanishing Plate Stiffnesses	
			Ordinary Material	Bimodulus Material
1	Aligned single ply or aligned parallel ply	0 or 90/90	$A_{16}, A_{26}, D_{16}, D_{26}$ , all $B_{ij}$	$A_{16}, A_{26}, B_{16}, B_{26}, D_{16}, D_{26}$
2	Cross-ply: $\bar{n} = \text{odd}^*$	0/90/0	$A_{16}, A_{26}, D_{16}, D_{26}$ , all $B_{ij}$	$A_{16}, A_{26}, B_{16}, B_{26}, D_{16}, D_{26}$
3	$\bar{n} = \text{even}$	0/90	$A_{16}, A_{26}, B_{16}, B_{26}, D_{16}, D_{26}$	$A_{16}, A_{26}, B_{16}, B_{26}, D_{16}, D_{26}$
4	Alternating angle-ply: $\bar{n} = \text{odd}$	30/-30/30	$A_{16}, A_{26}$ , all $B_{ij}$	None
5	$\bar{n} = \text{even}$	10/-10	$A_{16}, A_{26}, B_{11}, B_{12}, B_{22}, D_{16}, D_{26}$	None
6	SBAP	30/-30/-30/30	$A_{16}, A_{26}$ , all $B_{ij}$	None
7	Quasi-isotropic	0/-45/+45/90	$A_{16}, A_{26}, D_{16}, D_{26}$	None
8	$0/\pm \theta$	-30/0/30	$A_{16}, A_{26}, B_{11}, B_{12}, B_{22}, D_{16}, D_{26}$	None
9	$(0/\pm \theta)_s$	-30/30/0/30/-30	$A_{16}, A_{26}$ , all $B_{ij}$	None

The remainder of this discussion is limited to the particular bimodulus-material model originated by Bert (14,15) and known as the symmetric fiber-governed-compliance model. This model can be expressed mathematically as follows:

$$\{\epsilon_j\} = [S_{ji}]\{\sigma_i\} \quad (5)$$

where the compliance takes on different sets of values as follows:

$$[S_{ji}] = \begin{cases} [S_{ji}]^{(t)} & \text{if } \sigma_f \geq 0 \\ [S_{ji}]^{(c)} & \text{if } \sigma_f < 0 \end{cases} \quad (6)$$

Here  $\sigma_f$  denotes the fiber stress along its axis.

\* The symbol  $\bar{n}$  denotes the total number of layers in the laminate.

To implement the material model in plate theory, we introduce a third subscript ( $k$ ) to the  $Q_{ij}$  plane-stress stiffness matrix in which  $k=1,2$  denote tension and compression properties, respectively. Then by invoking the well-known Voigt hypothesis of uniformity of fiber-direction strain in the fibers and matrix of each respective individual layer, one obtains the following criteria for use of the appropriate  $Q_{ijk}$ :

$$[Q_{ijk}] = \begin{cases} [Q_{ij1}] & \text{if } \epsilon_f \geq 0 \\ [Q_{ij2}] & \text{if } \epsilon_f < 0 \end{cases} \quad (7)$$

where  $\epsilon_f$  denotes the fiber-direction strain at any arbitrary point. The locus of points at which  $\epsilon_f = 0$  is traditionally called the neutral surface.

Depending upon the plate geometry, boundary conditions, and loading, even in the case of a single-layer plate, the neutral surface in general is not a horizontal plane surface. Since the properties ( $Q_{ijk}$ ) depend upon the sign of  $\epsilon_f$  and the plate stiffnesses ( $A_{ij}, B_{ij}, D_{ij}$ ) upon the  $Q_{ijk}$  as in equations (3), it is clear that in general a bimodulus plate is nonhomogeneous in its plane, i.e., the plate stiffnesses depend upon position ( $x, y$ ). It is noted that, unlike a plate with linearly tapering thickness, here the plate stiffness in general is not only not a simple function of position, its functional form is not even known a priori. In this regard, there is a qualitative analogy between a bimodulus problem and an elastoplastic problem, for which the elastoplastic boundary is unknown a priori.

From the above discussion, it is clear that in order for a finite element to be applicable to any arbitrary combination of planform geometry, boundary conditions, and loading for a bimodulus laminated plate, it must have a full array of plate stiffnesses (none zero).

#### SMALL DEFLECTIONS OF THIN, CLAMPED ELLIPTIC PLATES

In studying structural mechanics problems involving new classes of materials, it is often quite instructive to obtain closed-form solutions for certain special cases. Then there are no questions regarding numerical approximations, convergence, etc. In the case of plates laminated of ordinary materials, the closed-form solutions due to Kicher (16) for elliptic plates and to Whitney and Leissa (11) for rectangular plates are most outstanding. Thus, it was decided to investigate the applicability of these forms of solutions to plates laminated of bimodulus materials.

Kicher's solution (16) is the closed-form solution for a uniformly-loaded, clamped-edge elliptic plate of cross-ply construction with an even number of layers (Case 3 in Table 2). The form of this solution is

$$\begin{aligned} u &= u_0 [1 - (x/a)^2 - (y/b)^2] (x/a) ; & v &= v_0 [1 - (x/a)^2 - (y/b)^2] (y/b) \\ w &= w_0 [1 - (x/a)^2 - (y/b)^2]^2 \end{aligned} \quad (8)$$

Here the displacement coefficients are readily determined by direct substitution into governing equations (1).

In view of the Kirchhoff hypothesis, upon which the present theory is based, the  $x$ -direction normal strain at any arbitrary location ( $x, y, z$ ) is

$$\epsilon_1 = u_{,x} - z w_{,xx} \quad (9)$$

Thus, if one sets  $\epsilon_1 = 0$  and solves for  $z$ , one obtains the neutral surface position  $z_{nx}$  associated with the normal strain in the  $x$  direction. It can readily be shown that when this procedure is applied to the Kicher displacements, equations (8), the resulting expression for  $z_{nx}$  is independent of  $x$  and  $y$ .



This means that for this particular problem, the plate stiffnesses are uniform throughout the plate surface of a single-layer bimodulus plate provided that the fibers are aligned in the x direction. (A similar conclusion can be reached for  $Z_{ny}$  and thus, a single-layer bimodulus plate with the fibers in the y direction.)

For the single-layer plate, it can be shown (8) that the plate stiffnesses depend upon  $Z (=z_n/h)$  in the following manner:

$$\begin{aligned} A_{ij}/h &= (1/2)(Q_{ij1} + Q_{ij2}) + (Q_{ij2} - Q_{ij1})Z \\ B_{ij}/h^2 &= (1/8)(Q_{ij1} - Q_{ij2}) + (Q_{ij2} - Q_{ij1})(Z^2/2) \\ D_{ij}/h^3 &= (1/24)(Q_{ij1} + Q_{ij2}) + (Q_{ij2} - Q_{ij1})(Z^3/3) \end{aligned} \quad (10)$$

The only thing remaining is the determination of the neutral-surface location  $Z$ . In principle, an equation in  $Z$  can be obtained by combining equations (10) with the expression for  $u_0/w_0$  as required by solution of equations (1). Although the resulting expression is only cubic in  $Z$ , the algebraic structure of the expressions for the coefficients of the cubic is so lengthy that it was found to be more expedient computationally to obtain  $Z$  by direct iteration. From physical considerations, only one unique solution for  $Z$  is meaningful (i.e. real, with  $|Z| < 1/2$ ). In (8), numerical results were presented for a composite material with considerable bimodulus action, namely aramid-cord/rubber which has the properties listed in Table 3, taken from experimental data of Patel et al. (17). The exact bimodulus results were compared with those obtained by ordinary theory using average properties. The necessity of using bimodulus theory is clearly demonstrated as shown in Figures 1 and 2.

Table 3. Bimodulus Properties of Aramid Tire-Cord/Rubber Composite, Reduced from Experimental Data in (17)

Property	Units	Tension	Compression
Major Young's modulus	MPa	3,580	12.0
Transverse Young's modulus	MPa	9.09	12.0
Inplane shear modulus	MPa	3.70 (3.83)	3.70 (3.83)
Major in-plane Poisson's ratio	-	0.416	0.205
Minor in-plane Poisson's ratio	-	0.01105	0.205

In view of the nature of Cases 2 and 3 in Table 2, one would expect the Kicher displacement functions to be applicable to an arbitrary cross-ply plate of bimodulus material. In (18), this is shown to be true and it is implemented for the "most unsymmetric" layer, namely the case of  $n=2$ . In this case, equations (10) are replaced by

$$\begin{aligned} A_{ij}/h &= (1/2)(Q_{ij11} + Q_{ij22}) + (Q_{ij21} - Q_{ij11})Z_x + (Q_{ij22} - Q_{ij12})Z_y \\ B_{ij}/h^2 &= (1/8)(Q_{ij11} - Q_{ij22}) + (Q_{ij21} - Q_{ij11})(Z_x^2/2) + (Q_{ij22} - Q_{ij12})(Z_y^2/2) \\ D_{ij}/h^3 &= (1/24)(Q_{ij11} + Q_{ij22}) + (Q_{ij21} - Q_{ij11})(Z_x^3/3) + (Q_{ij22} - Q_{ij12})(Z_y^3/3) \end{aligned} \quad (11)$$

Here the last subscript  $\lambda$  in stiffnesses  $Q_{ij\lambda\lambda}$  denotes the layer number (either layer 1 or 2) and the  $ijk$  subscripts have the same meaning as before.

In the case of cross-ply laminates, the elastic properties used for the layers having the fibers oriented in the x direction are determined by the x-direction neutral surface, while those for the other layers are determined by the y-direction neutral surface. Thus, to a certain extent a two-layer cross-



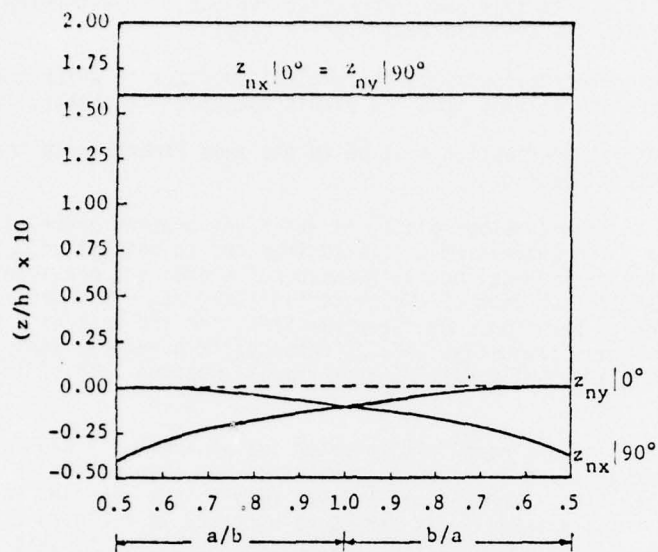


Fig. 1 Neutral-surface location vs. plate aspect ratio for single-layer aramid-rubber plate.

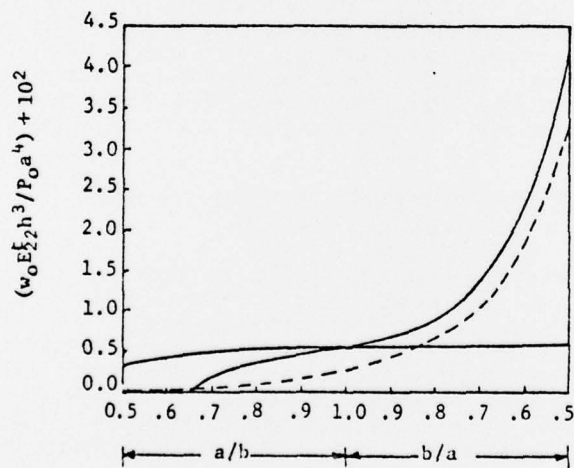


Fig. 2 Maximum deflection vs. plate aspect ratio for single-layer aramid-rubber plate. Bi-modulus solutions for fibers parallel ( $0^\circ$ ) and perpendicular ( $90^\circ$ ) to the x axis. Dashed line represents  $0^\circ$  case based on average of tension and compression properties.

ply bimodulus laminate is like a four-layer, cross-ply ordinary laminate (with undetermined individual layer thicknesses). Thus, there arises a question as to the proper order of the tensile and compressive regions. To determine the correct order, the following criteria were used in (18):

1. For bending-producing loadings, the neutral surfaces to which the iteration procedure leads must lie within the plate thickness, i.e.  $|Z_y| < 1/2$ .
2. The maximum plate deflection must be in the same direction as the normal-pressure loading.

In the case of an aramid/rubber plate, it was found that an order of stress-state tension T or compression C (listed from top to bottom) of C/T/T/C resulted in a negative maximum deflection (upward for a downward pressure), while an order of C/T/C/T met both of the criteria listed above and thus was judged to be the correct solution. On the other hand, for the case of a polyester/rubber plate, it was found that C/T/C/T resulted in a neutral surface lying outside of the plate, while C/T/T/C gave results meeting both of the above criteria.

In attempting to find a closed-form solution for an angle-ply bimodulus plate, the second author found a closed-form solution for an arbitrarily laminated ordinary laminate (19). However, it did not result in a constant value of neutral surface and thus, apparently it cannot be extended to a closed-form solution of the angle-ply bimodulus plate. Alternate approaches to this problem are currently being investigated.

#### NONLINEAR DEFLECTIONS OF THIN, CLAMPED ELLIPTIC PLATES

The geometrically nonlinear midplane strain-displacement relations originated by von Karman were incorporated into thin, laminated, anisotropic plate theory by Whitney and Leissa (11), although they did not solve the resulting nonlinear partial differential equations. Although the Ritz-Galerkin and Rayleigh-Ritz (energy) methods are probably equally popular, in (9) it was elected to use a perturbation procedure first used for plates by Nash, Cooley (20) for clamped, elliptic isotropic plates under uniform pressure. The only change in the procedure is the additional iteration procedure (necessary to take into account the bimodulus stiffnesses) and the additional elastic constants due to the bimodulus action.

#### SMALL DEFLECTIONS OF RECTANGULAR PLATES INCLUDING THICKNESS-SHEAR DEFORMATION

It has been demonstrated (21) that fiber-reinforced composite materials exhibit much larger thickness shear effects than do plates having the same geometry but constructed of homogeneous, isotropic materials. The explanation for this is due to the relatively low thickness shear moduli relative to the in-plane moduli. The numerous theories including these effects for laminated anisotropic plates were reviewed in (12). It suffices here to mention that the two most widely used of these theories are those due to Yang et al. (22) and Whitney and Pagano (23). It was shown in (24) that the latter theory is more accurate.

In (25), the modal shapes used by (26) for free vibration and buckling analyses of shear deformable cross-ply rectangular plates are used to analyze the static deflection of similar plates constructed of bimodulus composite materials and subjected to a sinusoidally distributed normal pressure. Again, as in the case of the elliptic plate problem discussed in the preceding section, the two criteria for constancy of the neutral-surface position are satisfied, and thus the solutions are exact, closed-form solutions.

# FINITE-ELEMENT FORMULATION FOR PLATES INCLUDING TRANSVERSE SHEAR DEFORMATION

While considerable effort has been expended in the finite-element analysis of isotropic plates, only limited investigations of laminated anisotropic plates can be found in the literature. Pryor and Barker (27), and Barker, Lin and Dara (28) used the conventional displacement finite-element method to analyze thick laminated plates. The element has seven degrees of freedom (three displacements, two rotations, and two shear rotations) per node. Exploiting the symmetries exhibited by anisotropic plates, Noor and Mathers (29-31) studied the effects of shear deformation and anisotropy on the accuracy and convergence of several shear-flexible displacement finite-element models based on a form of Reissner's plate theory. The analysis was limited to symmetrically laminated cross-ply plates and the element used involved 80 degrees of freedom per element. The conventional finite element, when applied to relatively thick laminated plates, either has failed to predict accurately the local deformations and stresses of a plate under bending or is too expensive to use due to large degrees of freedom involved for even relatively simple problems. Mau and Witmer (32), and Mau, Tong, and Pian (33) have employed the so-called hybrid-stress finite-element method to analyze composite plates including shear deformation. The hybrid elements have proven (see Gallagher (34)) to have some convergence problems, and in some cases they give erroneous results. Most recently, Panda and Natarajan (35) used, following Mawanya and Davies (36), the quadratic shell element of Ahmad, Irons and Zienkiewicz (37) with the same normal rotation through the thickness to claim improved accuracy over Mawanya and Davies (36). The 'thickness concept' mentioned in there is essentially the same as that used in the YNS theory (21). The authors were primarily concerned with the accuracy of the element, and no attempt was made to solve new problems for which there do not exist any closed-form, or exact solutions. In an effort to relax the continuity requirements on the shape functions, Reddy (38) recently devised a simple finite element and successfully tested the closed-form solutions of Bert and Chen (39). Only investigation that concerns with the finite element formulations of bimodulus materials is due to Crose, et al. (40). Here, following (38), we present the finite-element formulation.

Consider a plate of constant thickness  $h$  composed of thin anisotropic layers oriented at angles  $\theta_1, \theta_2, \dots$ . The origin of the coordinate system is located within the middle plane ( $x$ - $y$ ) with the  $z$ -axis being normal to the middle plane. The material of each layer is assumed to possess a plane of elastic symmetry parallel to the  $xy$ -plane. We shall denote the middle plane with  $\Omega$ .

The YNS theory is based on the following assumed displacement field:

$$\begin{aligned}\bar{u} &= u(x,y) + z\psi_x(x,y) \\ \bar{v} &= v(x,y) + z\psi_y(x,y) \\ w &= w(x,y)\end{aligned}\tag{12}$$

where  $\bar{u}$ ,  $\bar{v}$ , and  $w$  are the displacement components in the  $x$ ,  $y$  and  $z$ -directions, respectively,  $u$  and  $v$  are the in-plane (stretching) displacements of the middle plane, and  $\psi_x$  and  $\psi_y$  are the slope functions.

The equations of motion associated with the YNS theory are,

$$\begin{aligned}\frac{\partial N_1}{\partial x} + \frac{\partial N_6}{\partial y} &= 0 \\ \frac{\partial N_6}{\partial x} + \frac{\partial N_2}{\partial y} &= 0\end{aligned}$$

$$\begin{aligned}\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} &= p \\ \frac{\partial M_1}{\partial x} + \frac{\partial M_6}{\partial y} - Q_x &= 0 \\ \frac{\partial M_6}{\partial x} + \frac{\partial M_2}{\partial y} - Q_y &= 0\end{aligned}\quad (13)$$

where  $p = p(x, y)$  is the transversely distributed load, and  $N_i$ ,  $M_i$ ,  $Q_x$ , and  $Q_y$  are the stress resultants per unit length,

$$(N_1, N_2, N_6) = \int_{-h/2}^{h/2} (\sigma_x, \sigma_y, \tau_{xy}) dz, \quad (Q_x, Q_y) = \int_{-h/2}^{h/2} (\tau_{xz}, \tau_{yz}) dz \quad (14)$$

$$(M_1, M_2, M_6) = \int_{-h/2}^{h/2} (\sigma_x, \sigma_y, \tau_{xy}) z dz \quad (15)$$

A thin orthotropic material with provision for a shift of the neutral surface (due to different properties in tension and compression) has the following plate constitutive relations:

$$\begin{Bmatrix} N_1 \\ N_2 \\ Q_y \\ Q_x \\ N_6 \\ M_1 \\ M_2 \\ M_6 \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & 0 & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & 0 & 0 & A_{26} & B_{12} & B_{22} & B_{21} \\ 0 & 0 & A_{44} & A_{45} & 0 & 0 & 0 & 0 \\ 0 & 0 & A_{45} & A_{55} & 0 & 0 & 0 & 0 \\ A_{16} & A_{26} & 0 & 0 & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & 0 & 0 & B_{16} & D_{11} & D_{12} & D_{11} \\ B_{12} & B_{22} & 0 & 0 & B_{26} & D_{12} & D_{22} & D_{16} \\ B_{16} & B_{26} & 0 & 0 & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} u_x \\ v_y \\ w_y + \psi_y \\ w_x + \psi_x \\ u_y + v_x \\ \psi_{x,x} \\ \psi_{y,y} \\ \psi_{x,y} + \psi_{y,x} \end{Bmatrix}, \quad k=1 \text{ or } 2 \quad (16)$$

where the material coefficients  $A_{ij}$ ,  $B_{ij}$  and  $D_{ij}$  are given by

$$\begin{aligned}(A_{ij}, B_{ij}, D_{ij}) &= \int_{-h/2}^{h/2} Q_{ij} (1, z, z^2) dz, \quad (i, j=1, 2, 6) \\ &= \sum_{k=1}^n \int_{z_k}^{z_{k+1}} Q_{ij1k} dz + \sum_{k=1}^n \int_{z_n}^{z_{k+1}} Q_{ij2k} dz\end{aligned}\quad (17)$$

Here  $Q_{ij1k}$  denote the stiffness coefficients in the plate coordinates of the  $k$ -th layer in tension ( $k=1$ ) or compression ( $k=2$ ), and  $z_n$  is the distance from the midsurface to the neutral plane (which is unknown a priori).

We assume, over each element  $\Omega_e$ , the same kind of interpolation for all of the variables,

$$u^e = \sum_{i=1}^n u_i^e N_i^e, \quad v^e = \sum_{i=1}^n v_i^e N_i^e, \quad \text{etc.} \quad (n=\text{nodes per element}) \quad (18)$$



where  $N_i^e$  are the element interpolation (or shape) functions, and  $u_i^e$ , and  $v_i^e$  are the nodal values of  $u^e$  and  $v^e$ , respectively. Substituting (18) into the first variation of the total potential energy associated with (13) and collecting the coefficients of the variations ( $\delta u_i$ ,  $\delta v_i$ , etc.) we obtain

$$[K^e](\Delta^e) = \{F^e\} \quad (19)$$

where  $\{\Delta^e\} = \{u_e\}, \{v_e\}, \{w_e\}, \{\psi_x^e\}, \{\psi_y^e\}$ , and  $K_{ij}^{ab}$  are given by

$$\begin{aligned} K_{ij}^{11} &= A_{11}S_{ij}^x + A_{16}(S_{ij}^{xy} + S_{ji}^{xy}) + A_{66}S_{ij}^y \\ K_{ij}^{12} &= A_{11}S_{ij}^x + A_{16}S_{ij}^x + A_{26}S_{ij}^y + A_{66}S_{ji}^{xy} \\ K_{ij}^{14} &= B_{11}S_{ij}^x + B_{16}(S_{ij}^{xy} + S_{ji}^{xy}) + B_{66}S_{ij}^y \\ K_{ij}^{15} &= B_{12}S_{ij}^{xy} + B_{16}S_{ij}^x + B_{26}S_{ij}^y + B_{66}S_{ji}^{xy} \\ K_{ij}^{22} &= A_{26}(S_{ij}^{xy} + S_{ji}^{xy}) + A_{22}S_{ij}^y + A_{66}S_{ij}^x \\ K_{ij}^{24} &= B_{16}S_{ij}^x + B_{66}S_{ij}^{xy} + B_{12}S_{ji}^{xy} + B_{26}S_{ij}^y \\ K_{ij}^{25} &= B_{26}(S_{ij}^{xy} + S_{ji}^{xy}) + B_{66}S_{ij}^x + B_{22}S_{ij}^y \\ K_{ij}^{33} &= A_{44}S_{ij}^x + \epsilon_2^2 S_{ij}^y + A_{45}(S_{ij}^{xy} + S_{ji}^{xy}) \\ K_{ij}^{34} &= A_{44}S_{ij}^{x0} + A_{45}S_{ij}^{y0}, \quad K_{ij}^{35} = A_{45}S_{ij}^{x0} + A_{55}S_{ij}^{y0} \\ K_{ij}^{44} &= D_{11}S_{ij}^x + D_{16}(S_{ij}^{xy} + S_{ji}^{xy}) + D_{66}S_{ij}^y + A_{45}S_{ij}^0 \\ K_{ij}^{45} &= D_{12}S_{ij}^{xy} + D_{66}S_{ji}^{xy} + D_{16}S_{ij}^x + D_{26}S_{ij}^y + A_{45}S_{ij}^0 \\ K_{ij}^{55} &= D_{26}(S_{ij}^{xy} + S_{ji}^{xy}) + D_{66}S_{ij}^x + D_{22}S_{ij}^y + A_{55}S_{ij}^0 \\ K_{ij}^{13} &= K_{ij}^{23} = 0, \quad S_{ij}^{\xi n} = \int_{\Omega^e} N_i N_{j,n} dx dy, \quad (\xi, n=0, x, y) \\ F_i^3 &= \int_{\Omega^e} p N_i dx dy, \quad F_i^1 = F_i^2 = F_i^4 = F_i^5 = 0 \end{aligned} \quad (21)$$

The element stiffness matrices are assembled in the usual manner, and boundary conditions of the problem are imposed before solving for  $\{\Delta\}$ . In the present study, the four-node quadrilateral element of the serendipity family is used. The element stiffness matrices for this element is of order 20x20.

Figure 3 shows the influence of the aspect ratio ( $b/a$ ) and side-to-thickness ratio ( $a/h$ ) on the location of neutral surfaces for a single-layer, isotropic, bimodulus, simply supported rectangular plate subjected to sinusoidal loading,

$$p = P_0 \sin(\pi x/a) \sin(\pi y/b)$$

The following elastic properties are used:

$$\begin{aligned} E_{11}^t &= 3.584 \text{ GPa}, \quad E_{11}^c = 1.792 \text{ GPa}, \quad E_{22}^t = E_{11}^t, \quad E_{22}^c = E_{11}^c \\ G_{12}^t &= G_{12}^c = 1.27 \text{ GPa}, \quad \nu_{12}^t = \nu_{21}^t = 0.4, \quad \nu_{12}^c = \nu_{21}^c = 0.2 \end{aligned}$$



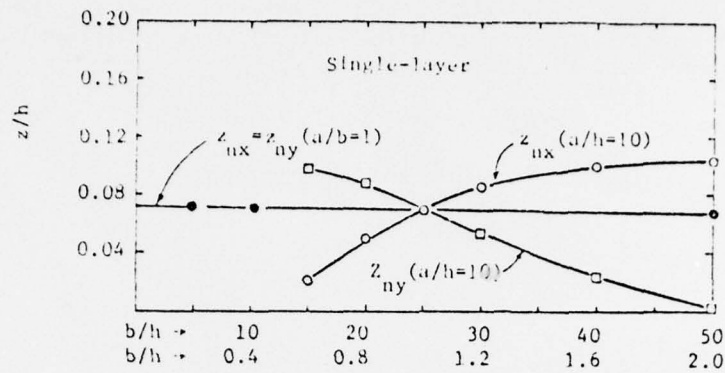


Fig. 3 Neutral-surface location vs. plate aspect ratio, and side-to-thickness ratio for single-layered rectangular plate under sinusoidal loading.

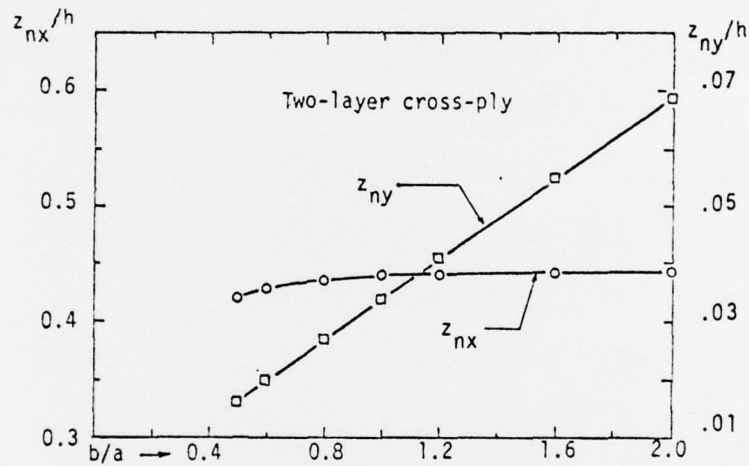


Fig. 4 Neutral-surface location vs. plate aspect ratio for two-layer, cross-ply ( $0^\circ/90^\circ$ ) square plate under sinusoidal loading.

Note that for  $b/a = 1$ , the neutral surfaces associated with x- and y-directions coincide (i.e.,  $z_{nx} = z_{ny}$ ).

Similar results are presented in Figures 4 and 5 for a two-layer, cross-ply ( $0^\circ/90^\circ$ ), rectangular plate under sinusoidal loading. The bimodulus properties used are the same as those listed in Table 3. Note from Fig. 4 that the neutral-surface location,  $z_{nx}$  is virtually unchanged for aspect ratio greater than 1, while the neutral-surface location,  $z_{ny}$  increases proportional to the aspect ratio. It should also be noted that the neutral surfaces do not coincide in the cross-ply case for  $b/a = 1$ .

Figure 6 shows the influence of the aspect ratio, and side-to-thickness ratio on the transverse deflection for single-layer, and two-layer cross-ply problems discussed above. The effect of thickness on the deflection is more pronounced than the effect of the aspect ratio.

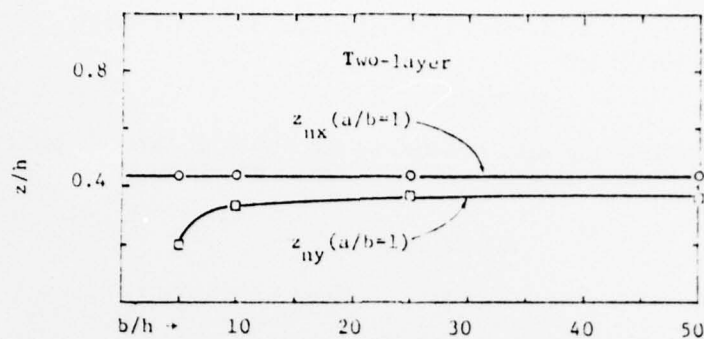


Fig. 5 Neutral-surface location vs. side-to-thickness ratio for two-layer, cross-ply ( $0^\circ/90^\circ$ ) rectangular plate under sinusoidal loading.

#### CONCLUDING REMARKS

A theory of plate bending for laminated, anisotropic bimodulus materials were presented. This theory is based on the fiber-governed symmetric compliance model for bimodulus materials.

Closed-form and finite-element solutions have been presented for selected problems involving different plate geometries, lamination arrangements, boundary conditions, both without and with thickness-shear deformation.

It was shown that even for single-layer bimodulus plates, the neutral-surface location may vary considerably from the geometric midplane, depending upon

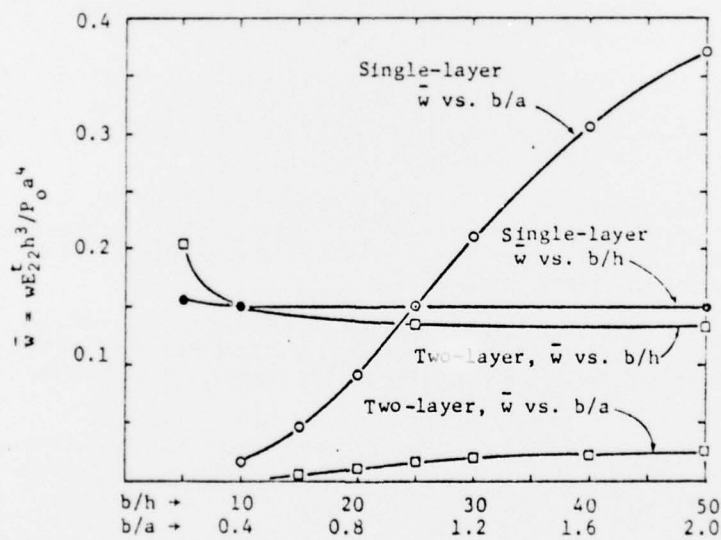


Fig. 6 Transverse deflection vs. plate aspect ratio, and side-to-thickness ratio for single-layer and two-layer cross-ply plates under sinusoidal loading.

the degree of bimodularity. Also, the plate deflection is significantly affected by the bimodulus action.

#### ACKNOWLEDGMENTS

The research reported here was sponsored by the Office of Naval Research. Analytical and computational assistance by S. K. Kincannon and W. C. Chao is gratefully acknowledged.

#### REFERENCES

- 1 Ambartsunyan, S.A., "The Axisymmetric Problem of a Circular Cylindrical Shell made of Material with Different Strength [Stiffnesses] in Tension and Compression", Izvestiya Akademiiy Nauk SSSR, Mekhanika, No. 4, 1965, pp. 77-85; Engl. translation, Rept. FTD-HT-23-1055-67, Dec. 1967; NTIS Document No. AD-675312.
- 2 Shapiro, G.S., "Deformation of Bodies with Different Tensile and Compressive Strengths [Stiffnesses]", Mechanics of Solids, Vol. 1, No. 2, 1966, pp. 85-86.
- 3 Kamiya, N., "Large Deflection of a Different Modulus Circular Plate", Journal of Engineering Materials and Technology, Transactions of ASME, Vol. 97H, No. 1, Jan. 1975, pp. 52-56.
- 4 Kamiya, N., "An Energy Method Applied to Large Elastic Deflection of a Thin Plate of Bimodulus Material", Journal of Structural Mechanics, Vol. 3, No. 3, 1975, pp. 317-329.
- 5 Kamiya, N., "Transverse Shear Effect in a Bimodulus Plate", Nuclear Engineering and Design, Vol. 32, No. 3, July 1975, pp. 351-357.
- 6 Ponomarev, B.V., "Flexure of Rectangular Plates for Nonlinear Elastic Materials of Asymmetric Behavior in Tension and Compression", Soviet Applied Mechanics, Vol. 4, No. 2, 1968, pp. 11-14.
- 7 Jones, R.M. and Morgan, H.S., "Bending and Extension of Cross-Ply Laminates with Different Moduli in Tension and Compression", Proceedings, AIAA/ASME/SAE 17th Structures, Structural Dynamics, and Materials Conference, King of Prussia, PA, May 5-7, 1976, pp. 158-167.
- 8 Bert, C.W. and Kincannon, S.K., "Bending-Extensional Coupling in Elliptic Plates of Orthotropic Bimodulus Material", Developments in Mechanics, Vol. 10 (Proceedings, 16th Midwestern Mechanics Conference), Kansas State University, Manhattan, KS, Sept. 19-21, 1979.
- 9 Kincannon, S.K. and Bert, C.W., "Nonlinear Analysis of Cross-Plyed Elliptical Plates with Different Moduli in Tension and Compression", to be presented at the American Society of Civil Engineers Annual Convention and Exposition, Atlanta, GA, Oct. 1979.
- 10 Reissner, E. and Stavsky, Y., "Bending and Stretching of Certain Types of Heterogeneous Anisotropic Elastic Plates", Journal of Applied Mechanics, Vol. 28, Transactions of ASME, Vol. 83E, No. 3, Sept. 1961, pp. 402-408.
- 11 Whitney, J.M. and Leissa, A.W., "Analysis of Heterogeneous Anisotropic Plates", Journal of Applied Mechanics, Vol. 36, Transactions of ASME, Vol. 91E, No. 2, June 1969, pp. 261-266.
- 12 Bert, C.W., "Analysis of Plates", Structural Design and Analysis, Part I, C.C. Chamis, ed., Academic Press, New York, 1974, pp. 149-206.

- 13 Bert, C.W., "Optimal Design of a Composite-Material Plate to Maximize its Fundamental Frequency", Journal of Sound and Vibration, Vol. 50, No. 2, Jan. 22, 1977, pp. 229-237.
- 14 Bert, C.W., "Models for Fibrous Composites with Different Properties in Tension and Compression", Journal of Engineering Materials and Technology, Transactions of ASME, Vol. 99H, No. 4, Oct. 1977, pp. 344-349.
- 15 Bert, C.W., "Recent Advances in Mathematical Modeling of the Mechanics of Bimodulus, Fiber-Reinforced Composite Materials", Proceedings, 15th Annual Meeting, Society of Engineering Science, Gainesville, FL, Dec. 1978, pp. 101-106.
- 16 Kicher, T.P., "The Analysis of Unbalanced Cross-Plyed Elliptic Plates Under Uniform Pressure", Journal of Composite Materials, Vol. 3, No. 3, July 1969, pp. 424-432.
- 17 Kincannon, S.K. and Bert, C.W., "Deflection of Uniformly Loaded Elliptic Plates of Cross-Plyed Bimodulus Material", unpublished manuscript, School of Aerospace, Mechanical, and Nuclear Engineering, University of Oklahoma, Norman, Oklahoma, 1979.
- 18 Patel, H.P., Turner, J.L., and Walter, J.D., "Radial Tire Cord-Rubber Composites", Rubber Chemistry and Technology, Vol. 49, No. 4, Sept.-Oct. 1976, pp. 1095-1110.
- 19 Bert, C.W., "Closed-Form Solution of an Arbitrarily Laminated, Anisotropic Thin Elliptic Plate Under Uniform Pressure", unpublished manuscript, 1979.
- 20 Nash, W.A. and Cooley, I.D., "Large Deflections of a Clamped Elliptical Plate Subjected to Uniform Pressure", Journal of Applied Mechanics, Vol. 26, Transactions of ASME, Vol. 81E, 1959, pp. 297-293.
- 21 Roze, A.V., "Bending of Plates Made of Oriented Glass-Reinforced Plastics", Polymer Mechanics, Vol. 1, No. 3, May-June 1965, pp. 91-97.
- 22 Yang, P.C., Norris, C.H., and Stavsky, Y., "Elastic Wave Propagation in Heterogeneous Plates", International Journal of Solids and Structures, Vol. 2, 1966, pp. 665-684.
- 23 Whitney, J.M. and Pagano, N.J., "Shear Deformation in Heterogeneous Anisotropic Plates", Journal of Applied Mechanics, Vol. 37, Transactions of ASME, Vol. 92E, No. 4, Dec. 1970, pp. 1031-1036.
- 24 Wang, A.S.D. and Chou, P.C., "A Comparison of Two Laminated Plate Theories", Journal of Applied Mechanics, Vol. 39, Transactions of ASME, Vol. 94E, No. 2, June 1972, pp. 611-613.
- 25 Bert, C.W. and Reddy, J.N., "Bending of a Moderately Thick, Cross-Ply Rectangular Plate Laminated of Fiber-Reinforced Bimodulus Composite Material", unpublished manuscript, 1979.
- 26 Sinha, P.K. and Rath, A.K., "Vibration and Buckling of Cross-Ply Laminated Circular Cylindrical Panels", Aeronautical Quarterly, Vol. 26, No. 3, Aug. 1975, pp. 211-218.
- 27 Pryor, C.W., Jr. and Barker, R.M., "A Finite Element Analysis Including Transverse Shear Effects for Applications to Laminated Plates", AIAA Journal, Vol. 9, No. 5, 1971, pp. 912-917.
- 28 Barker, R.M., Lin, F.T., and Dara, J.R., "Three-Dimensional Finite-Element Analysis of Laminated Composites", National Symposium on Computerized



Structural Analysis and Design, George Washington University, May, 1972.

29 Noor, A.K. and Mathers, M.D., "Shear-Flexible Finite Element Models of Laminated Composite Plates and Shells", NASA TN D-8044, 1975.

30 Noor, A.K. and Mathers, M.D., "Anisotropy and Shear Deformation in Laminated Composite Plates", AIAA Journal, Vol. 14, No. 2, 1976, pp. 282-285.

31 Noor, A.K. and Mathers, M.D., "Finite Element Analysis of Anisotropic Plates", International Journal for Numerical Methods in Engineering, Vol. 11, 1977, pp. 289-307.

32 Mau, S.T. and Witmer, E.A., "Static, Vibration, and Thermal Stress Analyses of Laminated Plates and Shells by the Hybrid-Stress Finite Element Method with Transverse Shear Deformation Effects Included", Aeroelastic and Structures Research Laboratory, Report ASRL TR 169-2, Department of Aeronautics and Astronautics, MIT, Cambridge, MA, 1972.

33 Mau, S.T., Tong, P., and Pian, T.H.H., "Finite Element Solutions for Laminated Thick Plates", Journal of Composite Materials, Vol. 6, 1972, pp. 304-311.

34 Gallagher, R.H., "Significant Advances in Finite Elements Over the Last Three Years", Proceedings, Conference on Mathematics of Finite Elements and Applications III, Brunel University, Uxbridge, England, 1978.

35 Panda, S.C. and Natarajan, R., "Finite Element Analysis of Laminated Composite Plates", International Journal for Numerical Methods in Engineering, Vol. 14, 1979, pp. 69-79.

36 Mawenya, S.A. and Davies, J.D., "Finite Element Bending Analysis of Multilayer Plates", International Journal for Numerical Methods in Engineering, Vol. 8, 1974, pp. 215-225.

37 Ahmad, S., Irons, B.M., and Zienkiewicz, O.C., "Analysis of Thick and Thin Shell Structures by Curved Finite Elements", International Journal for Numerical Methods in Engineering, Vol. 2, 1970, pp. 419-451.

38 Reddy, J.N., "Free Vibration of Antisymmetric, Angle-Ply Laminated Plates Including Transverse Shear Deformation by the Finite Element Method", Journal of Sound and Vibration, Vol. 66, No. 2, 1979, to appear.

39 Bert, C.W. and Chen, T.L.C., "Effect of Shear Deformation on Vibration of Antisymmetric Angle-Ply Laminated Rectangular Plates", International Journal of Solids and Structures, Vol. 14, No. 6, 1978, pp. 465-473.

40 Crose, J.C. and Jones, R.M., "SAAS III: Finite Element Stress Analysis of Axisymmetric and Plane Solids with Different Orthotropic, Temperature-Dependent Material Properties in Tension and Compression", Report TR-0059 (S6816-53)-1, Aerospace Corporation, San Bernardino, CA, June 1971, NTIS Document No. AD-729188.



UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER (14) OU-AMNE-79-8 STR-2.7	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) (6) ANALYSES OF PLATES CONSTRUCTED OF FIBER-REINFORCED BIMODULUS COMPOSITE MATERIALS.	(9)	5. TYPE OF REPORT & PERIOD COVERED Technical Report, No. 2
7. AUTHOR(s) (10) J. N./Reddy and C. W./Bert	(13)	6. PERFORMING ORG. REPORT NUMBER N00014-78-C-0647
9. PERFORMING ORGANIZATION NAME AND ADDRESS School of Aerospace, Mechanical & Nucl. Engrg. University of Oklahoma Norman, Oklahoma 73019	(11)	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS NR 064-609
11. CONTROLLING OFFICE NAME AND ADDRESS Department of the Navy, Office of Naval Research Structural Mechanics Program (Code 474) Arlington, Virginia 22217		12. REPORT DATE June 1979
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) (12) 28 P.		13. NUMBER OF PAGES 18
		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report)  This document has been approved for public release and sale; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES  To be presented at the Symposium on Mechanics of Bimodulus Materials, ASME Winter Annual Meeting, New York, NY, Dec. 1979 and to be published in the symposium volume.		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Bimodulus materials, closed-form solutions, composite materials, fiber-reinforced materials, finite-element analysis, elliptic plates, laminated plates, rectangular plates.		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) To implement the structural application of the recently introduced fiber-governed symmetric compliance model for bimodulus composite materials, both classical closed-form and finite-element solutions are being developed. This paper summarizes the results obtained for deflection of single-layer orthotropic and two-layer, cross-ply plates of the following configurations and loadings: →		

DD FORM 1 JAN 73 1473

EDITION OF 1 NOV 65 IS OBSOLETE  
S/N 0102-014-6601

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

400 498

elt

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

1. Thin elliptic plates clamped on the boundary and subjected to uniform pressure.
2. Moderately thick rectangular plate freely supported on the boundary and subjected to sinusoidally distributed pressure.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

